hw set 8
sect 5.7
p404

1. $\int \sin^3 x \cos^2 x \, dx \qquad u = \cos x$

$= \int \sin x (1-\cos^2 x)\cos^2 x \, dx$

$= \int \sin x \cos^2 x \, dx - \int \sin x \cos^4 x \, dx$

$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

2. $\int_0^{\pi/2} \cos^5 x \, dx \qquad u = \sin x$

$= \int_0^{\pi/2} \cos x \left[\cos^2 x\right]^2 dx$

$= \int_0^{\pi/2} \cos x \left[1-\sin^2 x\right]^2 dx$

$= \int_0^{\pi/2} \cos (1 - 2\sin^2 x + \sin^4 x)\, dx$

$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x \Big|_0^{\pi/2}$

$= \left[1 - 2/3 + 1/5\right] = 8/15$

3. $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x \, dx \qquad u = \sin x$

$= \int_{\pi/2}^{3\pi/4} \sin^5 x \left[1-\sin^2 x\right] \cos x \, dx$

$= \frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} \Big|_{\pi/2}^{3\pi/4} \qquad \sin\frac{3\pi}{4} = \sqrt{\frac{1}{2}}, \quad \sin \pi/2 = 1$

$= \left[\frac{1}{6}\left(\frac{1}{2}\right)^{6/2} - \frac{1}{8}\left(\frac{1}{2}\right)^{8/2}\right] - \left[\frac{1}{6} - \frac{1}{8}\right]$

$= -11/384 = -.0286458333$

7. $\int \tan^3 x \sec \, dx \qquad u = \sec x$

$du = \sec x \tan x$

$= \int \left[\sec^2 x - 1\right] \tan x \sec x \, dx \qquad 1 + \tan^2 x = \sec^2 x$

$= \int u^2 - \int 1 = \frac{1}{3}u^3 - u$

$= \frac{1}{3}\sec^3 x - \sec x$

17. $\int \frac{x-9}{(x+5)(x-2)} dx = \int \frac{A}{x+5} + \frac{B}{x-2}$

$x - 9 = A(x-2) + B(x+5)$

$x=2: \quad -7 = 7B, \quad B = -1$

$x=-5: \quad -14 = -7A, \quad A = 2$

$= \int \frac{2}{x+5} + \frac{-1}{x-2} dx = 2\ln(x+5) - \ln(x-2)$

18. $\int_0^1 \frac{x-1}{x^2+3x+2} dx = \int \frac{A}{(x+2)} + \frac{B}{(x+1)}$

$x - 1 = A(x+1) + B(x+2)$

$x=-1, \quad -2 = B$

$x=-2, \quad -3 = -A, \quad A = 3$

$= 3\ln(x+2) - 2\ln(x+1) \Big|_0^1$

$= \left[3\ln(3) - 2\ln(2)\right] - \left[3\ln(2) - 2\ln(1)\right]$

$= 3\ln 3 - 5\ln 2 = -.1698990368$

20. $\int \frac{x^2+2x-1}{x^3-x} dx = \int \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$

$x^2 + 2x - 1 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$

$x=0, \quad -1 = -A, \quad A = 1$

$x=1, \quad 2 = 2C, \quad C = 1$

$x=-1, \quad -2 = 2B, \quad B = 1$

$= \ln x - \ln(x+1) + \ln(x-1)$

21. $\int \frac{10}{(x-1)(x^2+9)} dx = \int \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$

$10 = A(x^2+9) + Bx(x-1) + C(x-1)$

$x=0, \quad 10 = 9A - C = 9 - C, \quad \boxed{C=-1}$

$x=1, \quad 10 = 10A, \quad A = 1$

$x=-1, \quad 10 = 10A + 2B - 2C$

$10 = 10 + 2B + 2$

$-2 = 2B, \quad B = -1$

$= \ln(x-1) + \int \frac{-x-1}{x^2+9} = \ln(x-1) + \int \frac{-x}{x^2+9} - \frac{1}{x^2+9}$

$= \ln(x-1) + -\frac{1}{2}\ln(x^2+9) - \frac{1}{3}\arctan\left(\frac{x}{3}\right)$

28. $\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx$

$$\begin{array}{r|l} & x+1 \\ \hline x^2-x-6 & x^3+0x^2-4x-10 \\ & x^3-x^2-6x \\ \hline & x^2+2x-10 \\ & x^2-x-6 \\ \hline & 3x-4 \end{array}$$

$$\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx = \int_0^1 x+1+\frac{3x-4}{(x-3)(x+2)}\,dx = \int_0^1 x+1+\frac{A}{x-3}+\frac{B}{x+2}\,dx$$

$$= \frac{x^2}{2}+x+A\ln|x-3|+B\ln|x+2|\,\Big|_0^1$$

$$= [1/2+1+A\ln 2+B\ln 3]-[A\ln 3+B\ln 2]$$

$$= 3/2+(A-B)\ln 2+(B-A)\ln 3$$

$$\begin{cases} 3x-4 = A(x+2)+B(x-3) \\ x=3, \quad 5=5A, \quad A=1 \\ x=-2, \quad -10=-5B, \quad B=2 \end{cases}$$

$$= 3/2+-\ln 2+\ln 3 = 3/2+\ln(3/2)$$

29. $\int_9^{16} \frac{\sqrt{x}}{x-4}\,dx$ want $\boxed{u=\sqrt{x}}$, $du=\frac{1}{2\sqrt{x}}\,dx \Rightarrow 2\,du=\frac{dx}{\sqrt{x}}$

E.C.

$$= \int_9^{16} \frac{\sqrt{x}}{x-4}\cdot\frac{\sqrt{x}}{\sqrt{x}}\,dx = \int_9^{16} \frac{x}{x-4}\cdot\frac{dx}{\sqrt{x}}$$

$$= \int \frac{2u^2}{u^2-4}\,du = \int 2+\frac{8}{u^2-4}\,du$$

$$\begin{array}{r|l} & 2 \\ \hline u^2-4 & 2u^2 \\ & 2u^2-8 \\ \hline & 8=r \end{array}$$

$$= \int 2+\frac{A}{u-2}+\frac{B}{u+2}\,du$$

$$\begin{cases} 8 = A(u+2)+B(u-2) \\ u=2, \quad 8=4A, \quad A=2 \\ u=-2, \quad 8=-4B, \quad B=-2 \end{cases}$$

$$= \int 2+\frac{2}{u-2}-\frac{2}{u+2}\,du = 2u+2\ln(u-2)-2\ln(u+2)$$

$$= 2\sqrt{x}+2\ln(\sqrt{x}-2)-2\ln(\sqrt{x}+2)\,\Big|_9^{16}$$

$$= [8+2\ln 2-2\ln 6]-[6+2\ln 1-2\ln 5]$$

$$= 2+2(\ln 2-\ln 6+\ln 5) = 2+2\ln\left(\tfrac{10}{6}\right)$$

$$= 2+2\ln(5/3)$$